JHEP HYPER VERSION

RECEIVED: *June 12, 2000*, ACCEPTED: *June 19, 2000*

Note on small black holes in $AdS_p \times S^q$

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ABSTRACT: It is commonly believed that small black holes in $AdS_5 \times S^5$ can be described by the ten dimensional Schwarzschild solution. This requires that the self-dual five-form (which is non-zero in the background) does not fall through the horizon and cause the black hole to grow. We verify that this is indeed the case: there are static solutions to the five-form field equations in a ten dimensional Schwarzschild spacetime. Similar results hold for other backgrounds $AdS_p \times S^q$ of interest in supergravity.

KEYWORDS: AdS/CFT Correspondance, Black Holes.

JHEP06(2000)031

One of the most important consequences of the AdS/CFT correspondence [1, 2] is the claim that the formation and evaporation of black holes can be described by a standard unitary evolution. Since this claim is contrary to well known semiclassical arguments [3], it is worthwhile to carefully examine the ingredients which go into this conclusion. One such ingredient is the assumption that a small black hole in $AdS_5 \times S^5$ will behave just like a ten dimensional Schwarzschild black hole. Intuitively, this seems reasonable since for a large AdS radius, the local description should be approximately given by the corresponding flat ten-dimensional spacetime physics; in particular, a small black hole should be approximately described by the 10-D Schwarzschild solution (sufficiently near the black hole).

However, the supergravity solution also includes a non-zero five-form. Although this acts like a cosmological constant in solutions which are products of two five dimensional spaces, in general it contains dynamical degrees of freedom. Given our experience with previous 'no-hair' theorems, one might worry that a small black hole will cause the five-form to fall into the horizon. Even though the local energy density in the five-form is small, if this were the case, most small black holes would grow by classically absorbing the energy density of the five-form and not quantum mechanically evaporate.

We show below that this does not occur. There exist static solutions for a self-dual five-form in the background of a ten dimensional black hole which have the correct boundary conditions at infinity to match onto the $AdS_5 \times S^5$ solution. It is these boundary conditions which effectively stabilize the field and invalidate the 'no-hair' intuition. The five-form is distorted by the black hole, but does not cause it to grow. We also show that similar results hold for four-forms and seven-forms in the background of an 11-D Schwarzschild solution with the right boundary conditions to match onto $AdS_4 \times S^7$ and $AdS_7 \times S^4$.

We start by noting that for pure $AdS_5 \times S^5$, the solution (in global coordinates) is given by the metric

$$ds^2 = -\left(\frac{\rho^2}{R^2}+1\right) dt^2 + \frac{d\rho^2}{(\rho^2/R^2)+1} + \rho^2\, d\Omega_3^2 + d\chi^2 + R^2 \sin^2\frac{\chi}{R}\, d\Omega_4^2\,, \qquad (1)$$

where R is the radius of curvature, and the five-form field strength $\tilde{F}$ is the sum of the volume form on AdS_5 and on S^5, normalized so that $\int_{S^5}\tilde{F} = N$. To simplify the formulas below, we will work with the rescaled five-form $F \equiv (\pi^3 R^5/N)\tilde{F}$, so that F is just the sum of the volume forms[1]

$$F = -\rho^3\, dt\wedge d\rho\wedge d\Omega_3 + R^4 \sin^4\frac{\chi}{R}\, d\chi\wedge d\Omega_4\,. \qquad (2)$$

How does this solution change in the presence of a black hole? For a black hole with radius larger than R, we already know what this modification is: the metric

[1]We use $d\Omega_n$ to denote the volume n-form on unit S^n and $d\Omega_n^2$ to denote the metric on S^n.

on AdS_5 is replaced with the five-dimensional Schwarzschild-AdS solution and the metric on the S^5 is unchanged

$$ds^2 = -\left(\frac{\rho^2}{R^2} + 1 - \frac{\rho_0^2}{\rho^2}\right) dt^2 + \frac{d\rho^2}{(\rho^2/R^2) + 1 - (\rho_0^2/\rho^2)} + \rho^2 d\Omega_3^2 + d\chi^2 + R^2 \sin^2 \frac{\chi}{R} d\Omega_4^2 \,. \tag{3}$$

Since this change in the metric does not effect the volume form on AdS, the five-form field strength F remains the same. In particular, the self-duality condition is satisfied because only the combination $dt \wedge d\rho$ is present in this condition, so that the mass-dependence cancels out, and the "Bianchi identity" $dF = 0$ is independent of the metric. (It is clear that F remains smooth even at the horizon since the volume form on Schwarzschild-AdS is smooth there.)

For a small black hole the picture becomes much less clear. The black hole is localized on the S^5 as well as in the AdS_5 [4], so that the metric no longer factorizes. Hence we cannot just look for a lower dimensional solution with an effective cosmological constant. Finding the appropriate exact solution to the full 10-D Einstein five-form field equations seems intractable. Since the curvature near the horizon of a small black hole should be much larger than the field strength F, to a good approximation one can ignore the backreaction and treat the five-form as a test field on a fixed background spacetime. In this approximation, the metric satisfies the vacuum equations, and the unique static, spherically symmetric black hole solution is the ten-dimensional Schwarzschild metric. However, this approximation is consistent only if there exists a static solution for a test self-dual five-form in this background, with the right boundary conditions. These boundary conditions can be understood as follows.

Very far away from the black hole both the metric and the five-form should approach the forms given respectively by eqs. (1) and (2). Since the black hole is much smaller than R, these forms are valid even into the approximately flat region of small ρ and χ. We can identify this approximately flat region with the asymptotic region far from the Schwarzschild black hole. This then sets our boundary conditions "at infinity". To be more explicit, we first write the 10-D Schwarzschild solution in convenient coordinates in which the boundary conditions are easily posed while the required symmetries are still manifest. In particular, we want to use the 10-D radial coordinate (fixed by the area of S^8), but to split S^8 into S^3 and S^4, corresponding to the rotational SO(4) symmetry of AdS_5 and the remaining (unbroken) SO(5) rotational symmetry on S^5. This is achieved by using the coordinate transformation

$$\rho = r \sin\theta \,, \qquad \chi = r\cos\theta \,, \tag{4}$$

where $0 \leq \theta \leq \pi/2$. In these coordinates, the flat spacetime metric obtained from (1) in the limit $\rho, \chi \ll R$ takes the form

$$ds^2 \;=\; -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\Omega_3^2 + \cos^2\theta\, d\Omega_4^2\right) . \tag{5}$$

(The angular term in the parentheses is equivalent to $d\Omega_8^2$.) Similarly, the five-form field strength obtained from (2) and (4) in the limit $\rho, \chi \ll R$ takes the form

$$F = -r^3 \sin^4\theta \, dt\wedge dr\wedge d\Omega_3 - r^4 \sin^3\theta \cos\theta \, dt\wedge d\theta\wedge d\Omega_3 + \\ + r^4 \cos^5\theta \, dr\wedge d\Omega_4 - r^5 \sin\theta \cos^4\theta \, d\theta\wedge d\Omega_4 \,. \tag{6}$$

One can easily recheck that F is still closed and self-dual.

In these coordinates, the 10-D Schwarzschild metric is given by:

$$ds^2 = -f(r)\, dt^2 + f^{-1}(r)\, dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\Omega_3^2 + \cos^2\theta \, d\Omega_4^2\right) \tag{7}$$

with $f(r) \equiv 1 - (r_+/r)^7$. A general ansatz for the field strength with the required symmetries (namely F being static and spherically symmetric on S^3 and S^4) can be obtained by taking each of the four terms in (6) and multiplying by arbitrary functions of r and θ:

$$F = -g_1(r,\theta)\, r^3 \sin^4\theta \, dt\wedge dr\wedge d\Omega_3 - g_3(r,\theta)\, r^4 \sin^3\theta \cos\theta \, dt\wedge d\theta\wedge d\Omega_3 + \\ + g_2(r,\theta)\, r^4 \cos^5\theta \, dr\wedge d\Omega_4 - g_4(r,\theta)\, r^5 \sin\theta \cos^4\theta \, d\theta\wedge d\Omega_4 \,. \tag{8}$$

Our boundary conditions require that $g_i(r,\theta) \to 1$ as $r \to \infty$.[2] To determine the field strength (8) explicitly, we now impose the physical conditions that F is closed and self-dual (with respect to the black hole metric (7)).

We first eliminate two of the four arbitrary functions g_i appearing in (8) by imposing self-duality, $F = *F$. The volume form associated with (7) is simply given by

$$\varepsilon_{(10)} = r^8 \sin^3\theta \cos^4\theta \, dt\wedge dr\wedge d\theta\wedge d\Omega_3\wedge d\Omega_4 \,. \tag{9}$$

Correspondingly, the dual of F is

$$*F = -g_1(r,\theta)\, r^5 \sin\theta \cos^4\theta \, d\theta\wedge d\Omega_4 + g_3(r,\theta)\, \frac{1}{f(r)}\, r^4 \cos^5\theta \, dr\wedge d\Omega_4 - \\ - g_2(r,\theta)\, f(r)\, r^4 \sin^3\theta \cos\theta \, dt\wedge d\theta\wedge d\Omega_3 - g_4(r,\theta)\, r^3 \sin^4\theta \, dt\wedge dr\wedge d\Omega_3 \,. \tag{10}$$

Self-duality then requires $g_4 = g_1$ and $g_3 = f g_2$, so that (8) becomes

$$F = g_1(r,\theta) \left[-r^3 \sin^4\theta \, dt\wedge dr\wedge d\Omega_3 - r^5 \sin\theta \cos^4\theta \, d\theta\wedge d\Omega_4\right] + \\ + g_2(r,\theta) \left[-r^4 f(r) \sin^3\theta \cos\theta \, dt\wedge d\theta\wedge d\Omega_3 + r^4 \cos^5\theta \, dr\wedge d\Omega_4\right] . \tag{11}$$

The condition that F is non singular at the horizon requires that the arbitrary functions $g_1(r,\theta)$ and $g_2(r,\theta)$ are smooth at r_+. This can be easily seen by rewriting (11)

[2] In principle, terms of the form $\gamma_1(r,\theta)\, dt\wedge d\Omega_4$ and $\gamma_2(r,\theta)\, dr\wedge d\theta\wedge d\Omega_3$ would also be consistent with all the symmetries, and would satisfy the boundary conditions provided $\gamma_i(r,\theta) \to 0$ as $r \to \infty$. However, since F is closed, $\partial_r\gamma_1 = \partial_\theta\gamma_1 = 0$, which, along with the boundary condition $\gamma_1 \to 0$, requires that $\gamma_1(r,\theta) \equiv 0$. Self duality of F then forces $\gamma_2(r,\theta) \equiv 0$. Hence these terms will not arise, and the most general form of F will indeed be given by (8).

in the ingoing Eddington coordinates, $(v, r, \theta, \Omega_3, \Omega_4)$, which are regular at the horizon. Since $v \equiv t + r_*$, where r_* is defined by $dr_* \equiv dr/f(r)$, we can simply rewrite $dt = dv - \frac{dr}{f(r)}$. The field strength is then expressed as

$$
\begin{aligned}
F = g_1(r,\theta)\ &\left[-r^3\ \sin^4\theta\, dv\wedge dr\wedge d\Omega_3 - r^5\ \sin\theta\ \cos^4\theta\, d\theta\wedge d\Omega_4\right] + \\
&+ g_2(r,\theta)\left[-r^4\, f(r)\ \sin^3\theta\ \cos\theta\, dv\wedge d\theta\wedge d\Omega_3 + r^4\ \sin^3\theta\ \cos\theta\, dr\wedge d\theta\wedge d\Omega_3 + \right. \\
&\qquad\qquad \left. + r^4\ \cos^5\theta\, dr\wedge d\Omega_4\right]
\end{aligned} \tag{12}
$$

and we see that all the terms are smooth at r_+ if g_i are smooth at r_+.

We now require that F is closed, $dF = 0$. Since dF has two non-trivial components, proportional to $dt\wedge dr\wedge d\theta\wedge d\Omega_3$ and to $dr\wedge d\theta\wedge d\Omega_4$, we obtain two independent equations by setting each component to 0:

$$
r^3\,\partial_\theta(g_1\ \sin^4\theta) - \partial_r(r^4\, f\, g_2)\ \sin^3\theta\ \cos\theta = 0\,, \tag{13}
$$

$$
\partial_r(r^5\, g_1)\ \sin\theta\ \cos^4\theta + r^4\,\partial_\theta(g_2\ \cos^5\theta) = 0\,. \tag{14}
$$

We can simplify these partial differential equations further by separation of variables. By writing $g_i(r,\theta) \equiv g_i(r)\,\tilde{g}_i(\theta)$, the radial and angular parts decouple. By direct substitution, (13) becomes

$$
\frac{\tilde{g}_1'(\theta)\ \tan\theta + 4\tilde{g}_1(\theta)}{\tilde{g}_2(\theta)} = k = \frac{r\, f(r)\, g_2'(r) + 4f(r)\, g_2(r) + r\, f'(r)\, g_2(r)}{g_1(r)}\,, \tag{15}
$$

whereas (14) yields

$$
\frac{-\tilde{g}_2'(\theta)\ \cot\theta + 5\tilde{g}_2(\theta)}{\tilde{g}_1(\theta)} = l = \frac{r\, g_1'(r) + 5g_1(r)}{g_2(r)}\,, \tag{16}
$$

where k and l are arbitrary separation constants. These are, however, fixed by the boundary conditions: since $g_i(r)\,\tilde{g}_i(\theta) \to 1$ as $r \to \infty$, each function must approach a constant, which we can require to be one, as $r \to \infty$: i.e. $g_i(r) \to 1$ and $\tilde{g}_i(\theta) \to 1$. The latter requirement dictates that $\tilde{g}_i(\theta) = 1$, so that the angular part is trivial. This fixes the separation constants completely: $k = 4$ and $l = 5$. (We note that this is also self-consistently required by the radial parts of (15) and (16).)

Thus, we are left with the following coupled, linear, first order, ordinary differential equations for $g_1(r)$ and $g_2(r)$:

$$
g_1(r) = f(r)\, g_2(r) + \frac{1}{4}\, r\, \frac{d}{dr}\left(f(r)\, g_2(r)\right)\,, \tag{17}
$$

$$
g_2(r) = g_1(r) + \frac{1}{5}\, r\, \frac{d}{dr} g_1(r) \tag{18}
$$

with the asymptotic boundary conditions $g_i(r) \to 1$ as $r \to \infty$. Ordinarily, one would expect to be able to specify both g_1 and g_2 at $r = r_+$ and then integrate out to infinity. One could then hope to choose these two initial conditions to satisfy the two boundary conditions. However, (17) implies the following constraint at the horizon (using the fact that $f(r_+) = 0$):

$$g_1(r_+) = \frac{7}{4}\, g_2(r_+)\,, \tag{19}$$

so the solutions are determined by only one free parameter. Nevertheless, it is still possible to satisfy both boundary conditions. This is most easily seen by substituting (18) into (17) to obtain a decoupled, second order equation for $g_1(r)$:

$$f(r)\, g_1''(r) + \left(\frac{10}{r}\, f(r) + f'(r)\right) g_1'(r) + \left(\frac{20}{r^2}\,(f(r)-1) + \frac{5}{r}\, f'(r)\right) g_1(r) = 0\,. \tag{20}$$

The asymptotic form of this equation is

$$g_1''(r) + \frac{10}{r}\, g_1'(r) = 0\,, \tag{21}$$

so as $r \to \infty$, we have $g_1(r) \sim \text{const} + O(1/r^9)$. There is only a one parameter family of solutions to the second order equation (20) which are regular at the horizon since $f(r_+) = 0$ implies

$$g_1{}'(r_+) = -\frac{15}{7r_+}\, g_1(r_+)\,. \tag{22}$$

So given $g_1(r_+)$, we get a unique solution of the second order equation (20). We can clearly rescale $g_1(r_+)$ so that $g_1 \to 1$ at infinity. The function g_2 is then completely determined by (18), but fortunately it automatically satisfies the right boundary condition, $g_2 \to 1$ asymptotically. This shows that a solution satisfying all boundary conditions does exist.

Although we have not found the solution analytically, one can easily find it numerically.[3] A plot of the solution is shown in figure 1. We see that g_1 is enhanced and g_2 is slightly suppressed at the horizon, while both functions asymptote to the correct value, $g_i \to 1$.

Even though we have found static solutions for a test five-form field strength in a 10-D Schwarzschild background (given by (11)), F does not vanish at the horizon. So to ensure that the solution remains static when the backreaction is included, we need to check that there is no energy flux crossing the horizon. By the Raychaudhuri equation [5], the horizon area can remain constant only if $R_{ab}\, k^a\, k^b = 0$, where k^a denotes the null generators of the horizon, $k^a = (\partial/\partial t)^a$. Thus, a static configuration must satisfy $T_{ab}\, k^a k^b = 0$ at the horizon. One can easily show that this is indeed the case for our solution:

$$T_{ab}\, k^a k^b \propto k^a F_{acdem}\, k^b F_b{}^{cdem} \tag{23}$$

[3]However, we cannot integrate the solution directly from the horizon, since $g_1''(r_+)$ is undetermined there due to the factor of $f(r) = 0$ at $r = r_+$. Instead, we must obtain new "initial conditions" near the horizon, at $r = r_+ + \varepsilon$.

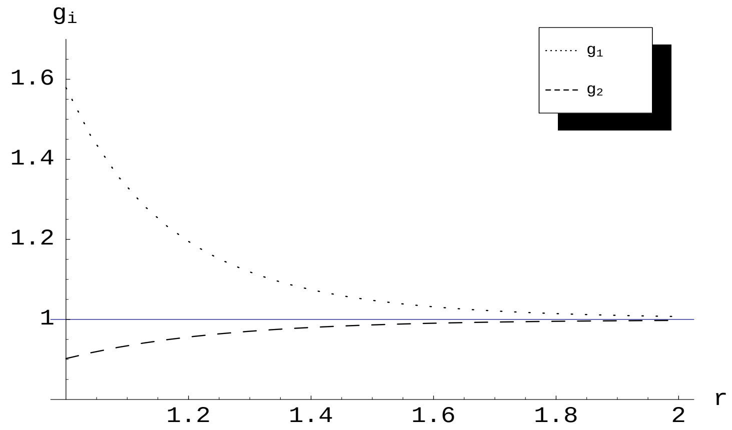

Figure 1: Solutions $g_1(r)$ (dotted line) and $g_2(r)$ (dashed line), and their asymptotic value of one (solid line) for a 10-D black hole with radius $r_+ = 1$.

and from (11), we have (in component notation)

$$k^a F_{acdem} \propto g_1(r)\, r^3 \sin^4\theta\, (dr)_{[c}(d\Omega_3)_{dem]} + \\ + g_2(r)\, r^4\, f(r) \sin^3\theta \cos\theta\, (d\theta)_{[c}(d\Omega_3)_{dem]}\,. \tag{24}$$

Contracting over $c, d, e,$ and m yields

$$k^a\, F_{acdem}\, k^b F_b{}^{cdem} \propto f(r)\, g_1^2(r) \sin^2\theta + f^2(r)\, g_2^2(r) \cos^2\theta \tag{25}$$

which clearly vanishes at the horizon, since $f(r_+) = 0$ and $g_i(r_+)$ remain finite. Hence

$$T_{ab}\, k^a k^b = 0 \tag{26}$$

is indeed satisfied at the horizon.

So far, we have considered a five-form field strength in the presence of a small 10-D black hole in asymptotically $AdS_5 \times S^5$ spacetime. We now check that the arguments of the preceeding section also apply to the other cases of interest for the AdS/CFT correspondence.

We start with the 11-D supergravity solutions $AdS_4 \times S^7$ and $AdS_7 \times S^4$. For conciseness, we combine these into the general case of $AdS_p \times S^q$, where $(p, q) = (4, 7)$ and $(7, 4)$. Here the logic of the argument is slightly different from the previous case, since dimensionally the field strength cannot be self-dual. Nonetheless, we shall see that the final differential equations are very similar to (17) and (18) (and are in fact identical if we set $p = q = 5$). This will allow us to apply the same arguments as above to prove the existence of a static solution satisfying the correct boundary conditions.

As in the preceeding discussion, we start with the metric in global AdS coordinates:

$$ds^2 = -\left(\frac{\rho^2}{R^2} + 1\right) dt^2 + \frac{d\rho^2}{(\rho^2/R^2) + 1} + \rho^2\, d\Omega_{p-2}^2 + d\chi^2 + (\alpha R)^2 \sin^2\frac{\chi}{\alpha R}\, d\Omega_{q-1}^2\,, \tag{27}$$

where α is a numerical constant, corresponding to the ratio of the size of the sphere to the size of AdS for the given supergravity solution ($\alpha = 1/2$ for $AdS_7 \times S^4$, and $\alpha = 2$ for $AdS_4 \times S^7$). The flat space approximation ($\rho, \chi \ll R$) of the metric and the corresponding volume form are given by

$$ds^2 = -dt^2 + d\rho^2 + \rho^2\, d\Omega_{p-2}^2 + d\chi^2 + \chi^2\, d\Omega_{q-1}^2\,, \tag{28}$$

$$\varepsilon_{p+q} = \rho^{p-2}\, \chi^{q-1}\, dt \wedge d\rho \wedge d\Omega_{p-2} \wedge d\chi \wedge d\Omega_{q-1}\,. \tag{29}$$

Hence, the p-form field strength and its q-form dual in this region are simply

$$F_{(p)} = -\rho^{p-2}\, dt \wedge d\rho \wedge d\Omega_{p-2}\,, \tag{30}$$

$$*F_{(q)} = \chi^{q-1}\, d\chi \wedge d\Omega_{q-1}\,. \tag{31}$$

Now, we use the change of coordinates (4) to write the $(p+q)$-dimensional Schwarzschild metric in the form

$$ds^2 = -f(r)\, dt^2 + f^{-1}(r)\, dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\Omega_{p-2}^2 + \cos^2\theta\, d\Omega_{q-1}^2\right)\,, \tag{32}$$

where $f(r) \equiv 1 - (r_+/r)^{p+q-3}$. Since the full $(p+q)$-dimensional volume form is independent of f, it can be obtained from (4) and (29)

$$\varepsilon_{p+q} = (-1)^{p-1}\, r^{p+q-2}\, \sin^{p-2}\theta\, \cos^{q-1}\theta\, dt \wedge dr \wedge d\theta \wedge d\Omega_{p-2} \wedge d\Omega_{q-1}\,. \tag{33}$$

Up till now, everything was just a simple generalization of the $AdS_5 \times S^5$ case. However, the general p-form in the presence of the localized black hole, which is consistent with all the symmetries now has only two arbitrary functions,

$$\begin{aligned} F_{(p)} = &-g_1(r,\theta)\, r^{p-2}\, \sin^{p-1}\theta\, dt \wedge dr \wedge d\Omega_{p-2} - \\ &- g_2(r,\theta)\, f(r)\, r^{p-1}\, \sin^{p-2}\theta\, \cos\theta\, dt \wedge d\theta \wedge d\Omega_{p-2} \end{aligned} \tag{34}$$

$g_1(r,\theta)$ and $g_2(r,\theta)$ are smooth everywhere and chosen such that they satisfy the simple flat space boundary condition[4] $g_1(r,\theta) \to 1$ and $g_2(r,\theta) \to 1$ as $r \to \infty$. (The other two terms which appeared in (8) for $F_{(5)}$ are not consistent with the dimensionality: they are q-forms rather than p-forms.) The dual q-form is then

$$*F_{(q)} = -g_1(r,\theta)\, r^q\, \sin\theta\, \cos^{q-1}\theta\, d\theta \wedge d\Omega_{q-1} + g_2(r,\theta)\, r^{q-1}\, \cos^q\theta\, dr \wedge d\Omega_{q-1}\,. \tag{35}$$

The differential equations for g_1 and g_2 are obtained by the condition that both the p-form field strength and its dual q-form must be closed, i.e. $dF_{(p)} = 0$ and $d{*F_{(q)}} = 0$. Canceling out the angular dependence (using the same separation of variables procedure as before) yields

$$g_1(r) = f(r)\, g_2(r) + \frac{1}{p-1}\, r\, \partial_r \left(f(r)\, g_2(r)\right)\,, \tag{36}$$

$$g_2(r) = g_1(r) + \frac{1}{q}\, r\, \partial_r g_1(r)\,. \tag{37}$$

[4]The function $f(r)$ was inserted into the second term for later convenience. (Note that $f(r) \to 1$ as $r \to \infty$, so the asymptotic boundary conditions remain uneffected.)

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Note that, as advertised, for $p = q = 5$, these equations are identical to (17) and (18) obtained above. The second order ODE for $g_1(r)$ is

$$f(r)\, g_1''(r) + \left(\frac{p+q}{r}\, f(r) + f'(r)\right) g_1'(r) + \\ + \left(\frac{q(p-1)}{r^2}\,(f(r)-1) + \frac{q}{r} f'(r)\right) g_1(r) = 0\,. \tag{38}$$

The asymptotic form of this equation,

$$g_1''(r) + \frac{p+q}{r}\, g_1'(r) = 0 \tag{39}$$

implies $g_1(r) \sim \text{const} + O(1/r^{p+q-1})$ as $r \to \infty$.

Solutions which are regular at the horizon are again determined by one parameter since

$$g_1{}'(r_+) = -\frac{q}{r_+}\,\left(\frac{q-2}{p+q-3}\right) g_1(r_+)\,. \tag{40}$$

One can choose this parameter so that $g_1 \to 1$ asymptotically. Then g_2 is uniquely determined by (37) and automatically satisfies the right boundary condition $g_2 \to 1$.

One can similarly show that the same conclusion will hold for another case of interest for the AdS/CFT duality, namely in IIB supergravity on $AdS_3 \times S^3 \times T^4$. In this case, the 4-torus decouples, and by a similar procedure as for $AdS_5 \times S^5$, we arrive at equations (36) and (37), with $p = q = 3$.

In all of the above cases one can easily verify that there is no flux of energy across the event horizon, $T_{ab}k^a k^b = 0$. (Note that although the stress tensor now has a non-zero trace, the term proportional to the metric does not contribute when contracted with the null vectors $k^a k^b$.) So the solutions will remain static when backreaction is included. Thus, we have shown that in all the relevant cases, a small black hole in $AdS_p \times S^q$ can indeed be approximated by a $(p+q)$-dimensional Schwarzschild solution. The Ramond-Ramond fields will be distorted, but will remain static and not cause the black hole to grow. In retrospect, it is perhaps not surprising that static solutions do exist, since they can be viewed as higher dimensional generalizations of a black hole in a background magnetic field [6].

It should be noted that the validity of the Schwarzschild approximation does not imply that all small black holes will Hawking evaporate. As noted in [7], if one fixes the total energy, the asymptotic AdS boundary conditions ensure that certain small black holes can be in stable equilibrium with their own Hawking radiation. However, sufficiently small black holes will still evaporate, so the AdS/CFT correspondence leads one to believe that this evaporation can be described by a unitary evolution.

Acknowledgments

This work was supported in part by NSF Grant PHY95-07065.

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JHEP06(2000)031